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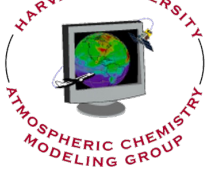


# Efficient computation of global high-resolution Jacobian matrices for analytical inversions of satellite observations of greenhouse gases

Dandan Zhang

with contributions from

Daniel J. Varon, Elizabeth W. Lundgren, Melissa P. Sulprizio, Nicholas Balasus, Lucas Estrada, and Daniel J. Jacob

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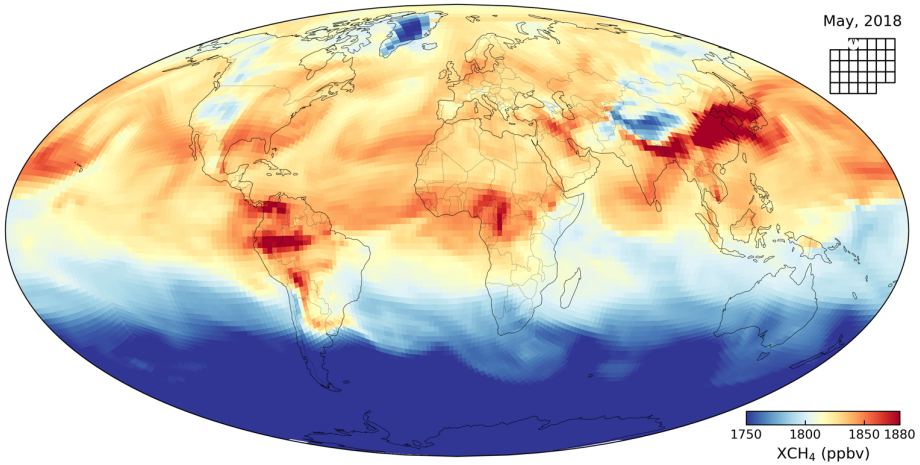
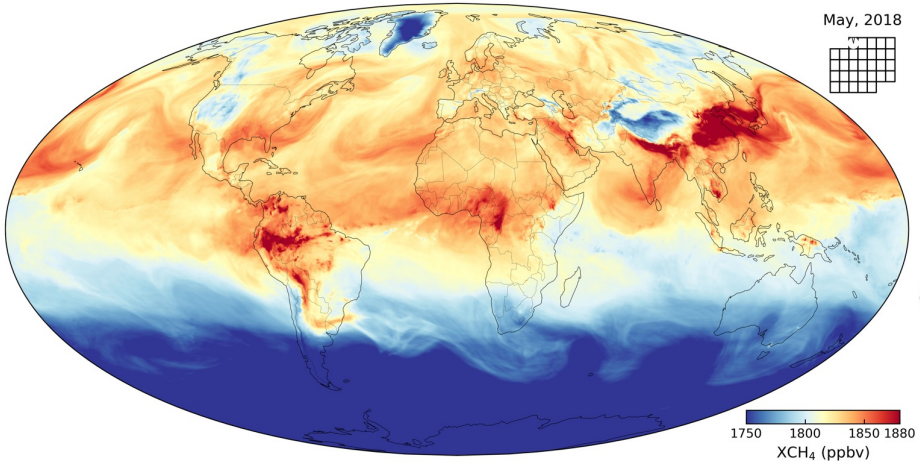
# Low-resolution forward model limits the capability to exploit growing high-resolution satellite observations in inverse analyses

≈ 200×200 km²




≈ 25×25 km²

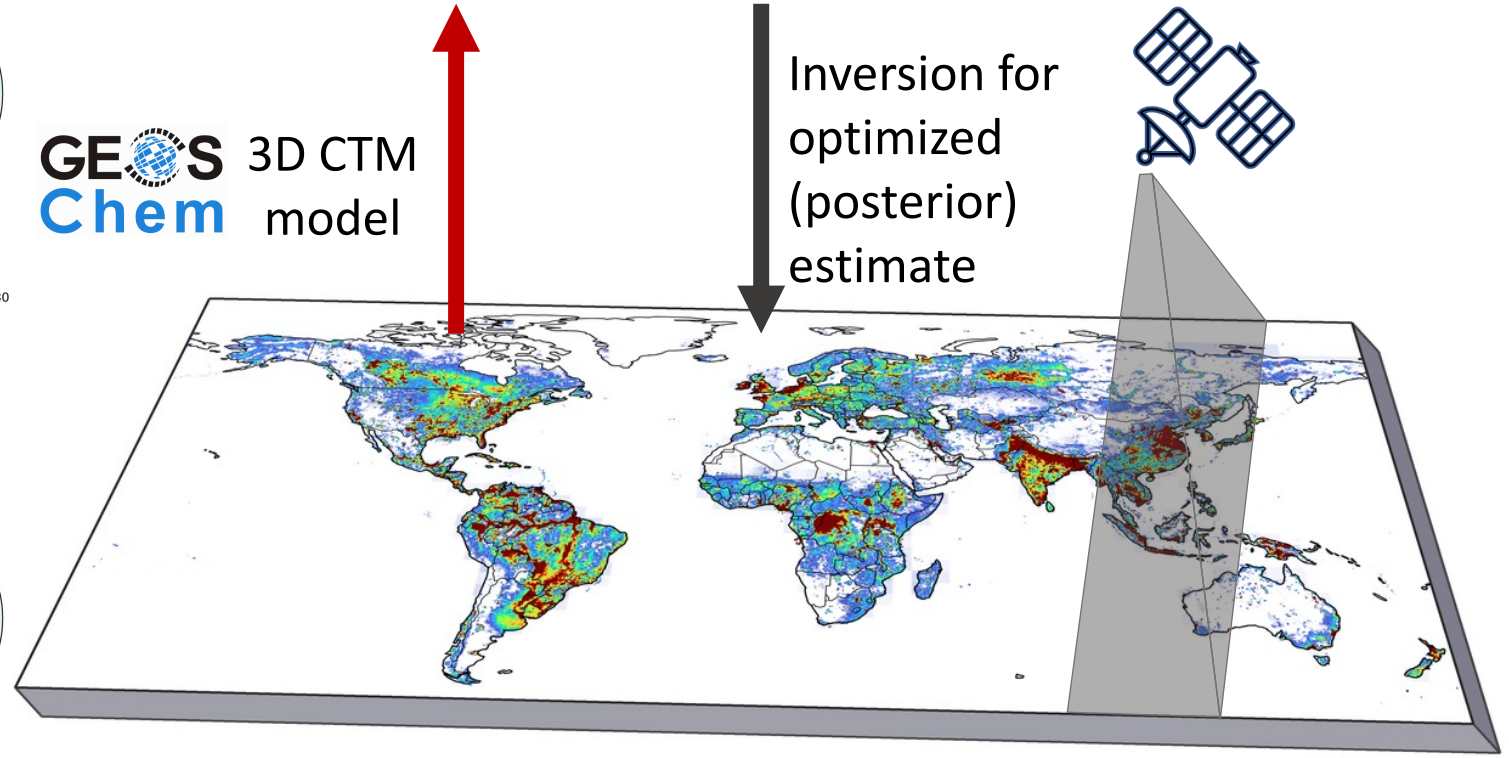



Global ≈ 200×200 km² ← Resolution degradation → Global 5.5×7 km²

Simulated concentrations ← compare → Satellite observations

GEOS Chem 3D CTM model

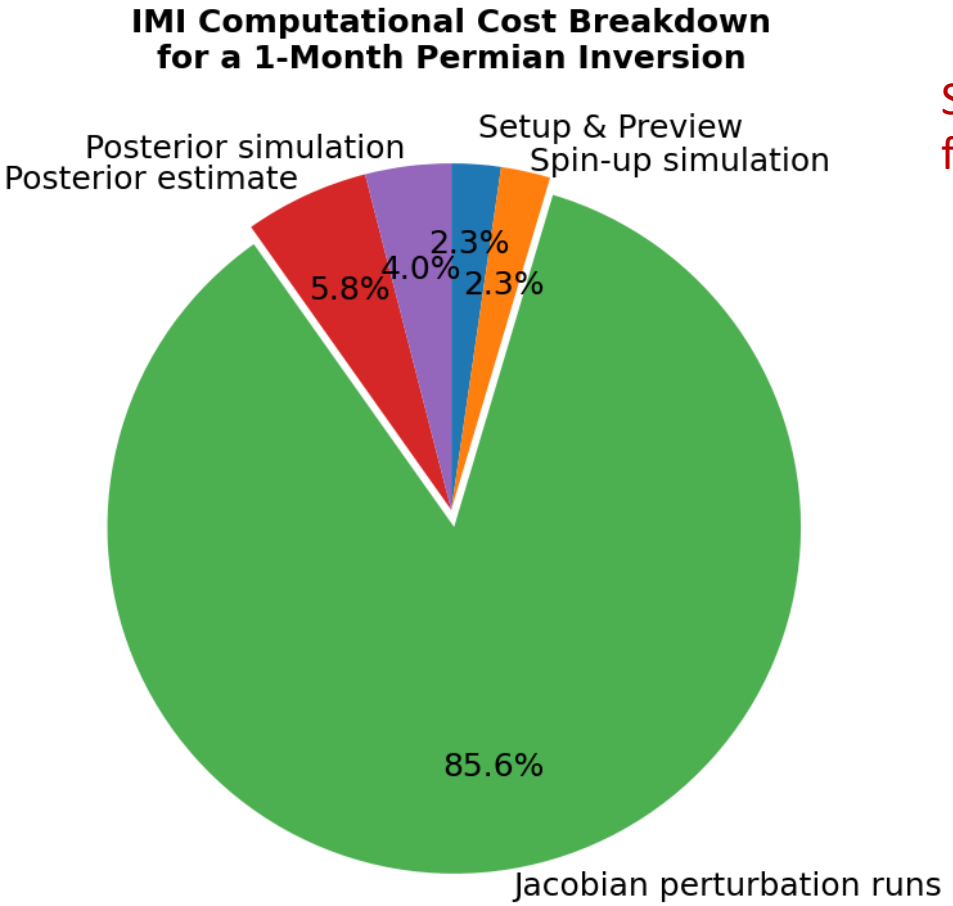
Inversion for optimized (posterior) estimate



Prior estimate from bottom-up emission inventories

# Analytical inversion with explicit Jacobians is computationally demanding

## Constructing Jacobian sensitivity dominates IMI computational cost




Ref: Varon et al., 2022, *Geosci. Model Dev.*

Jacobian sensitivity matrix (**J**): $[j_1, \cdots j_i, \cdots j_n]$

Concentration difference upon emission perturbation

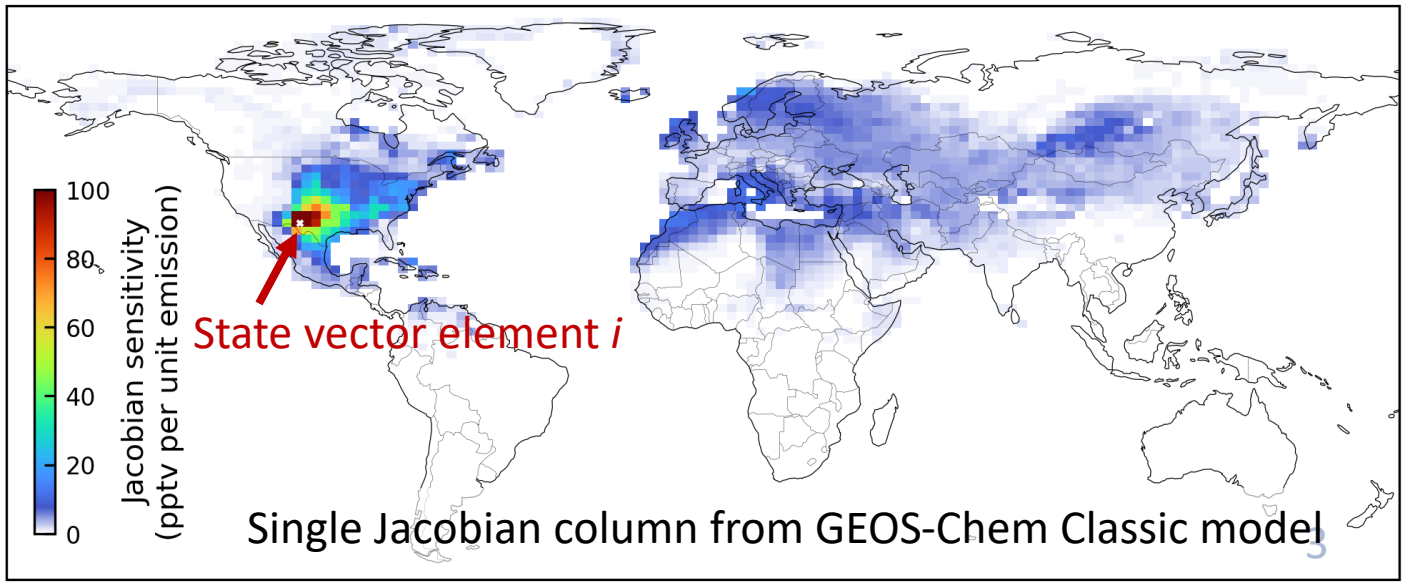
Single Jacobian column for state vector element *i* → $j_i = \frac{dy}{dx_i}$

$dy$ ← Concentration difference for all domain pixels (global/regional)

$dx_i$ ← Emission perturbation for single state vector element *i* (one pixel)

*n*: # of state vector elements

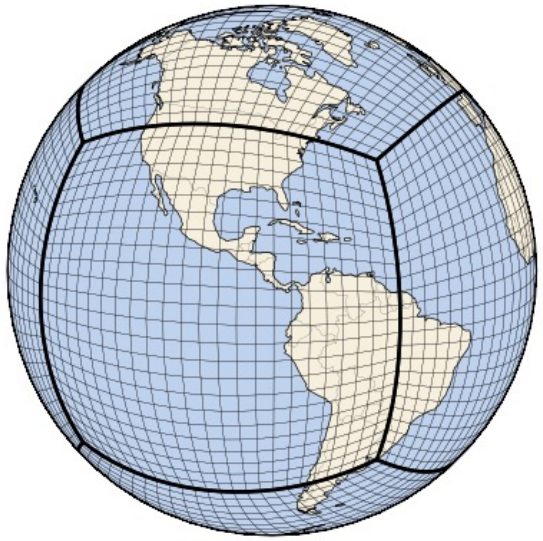
Global concentration response from single emission plume sampled at TROPOMI observations




Single Jacobian column from GEOS-Chem Classic model

# Solution: Global adaptive fine resolution enabled by stretched GCHP

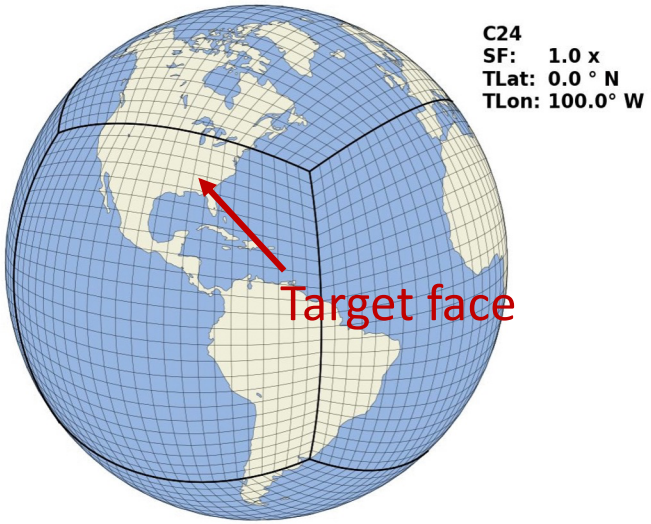
## Reduce computational cost for each state vector element by 100 times



GCHP: multi-node parallelization
Global fine-resolution capability
but computationally demanding

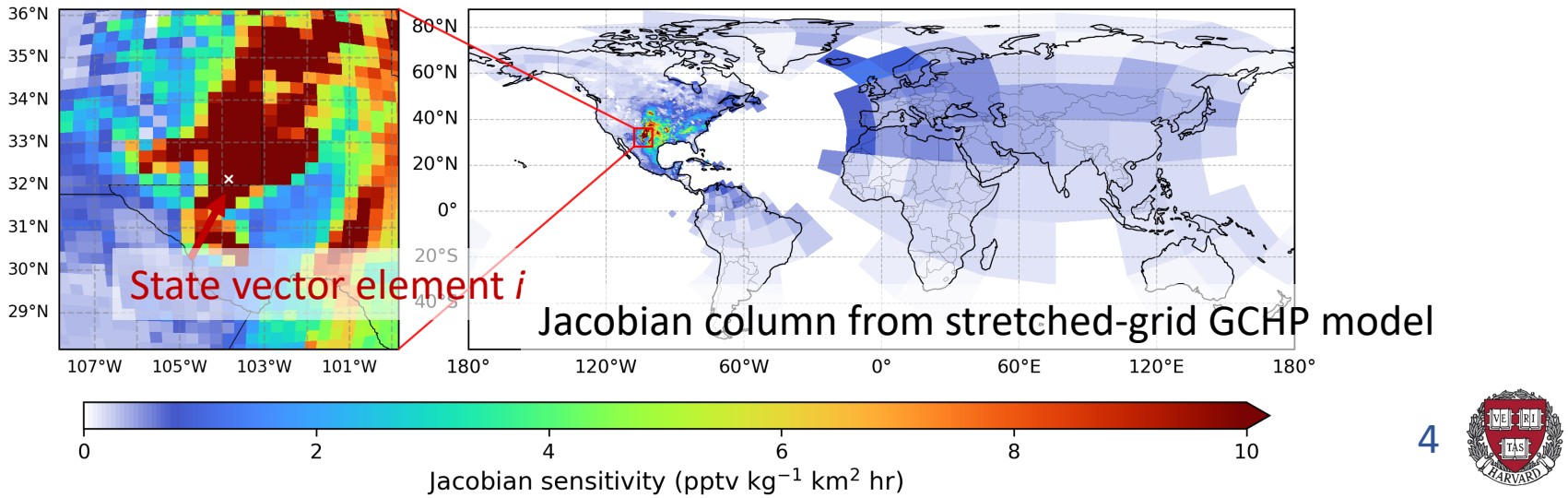
Stretched-grid GCHP simulation

- Computational cost of a coarse simulation
- Near-field finer resolution
- Global mass conservation & OH sink optimization



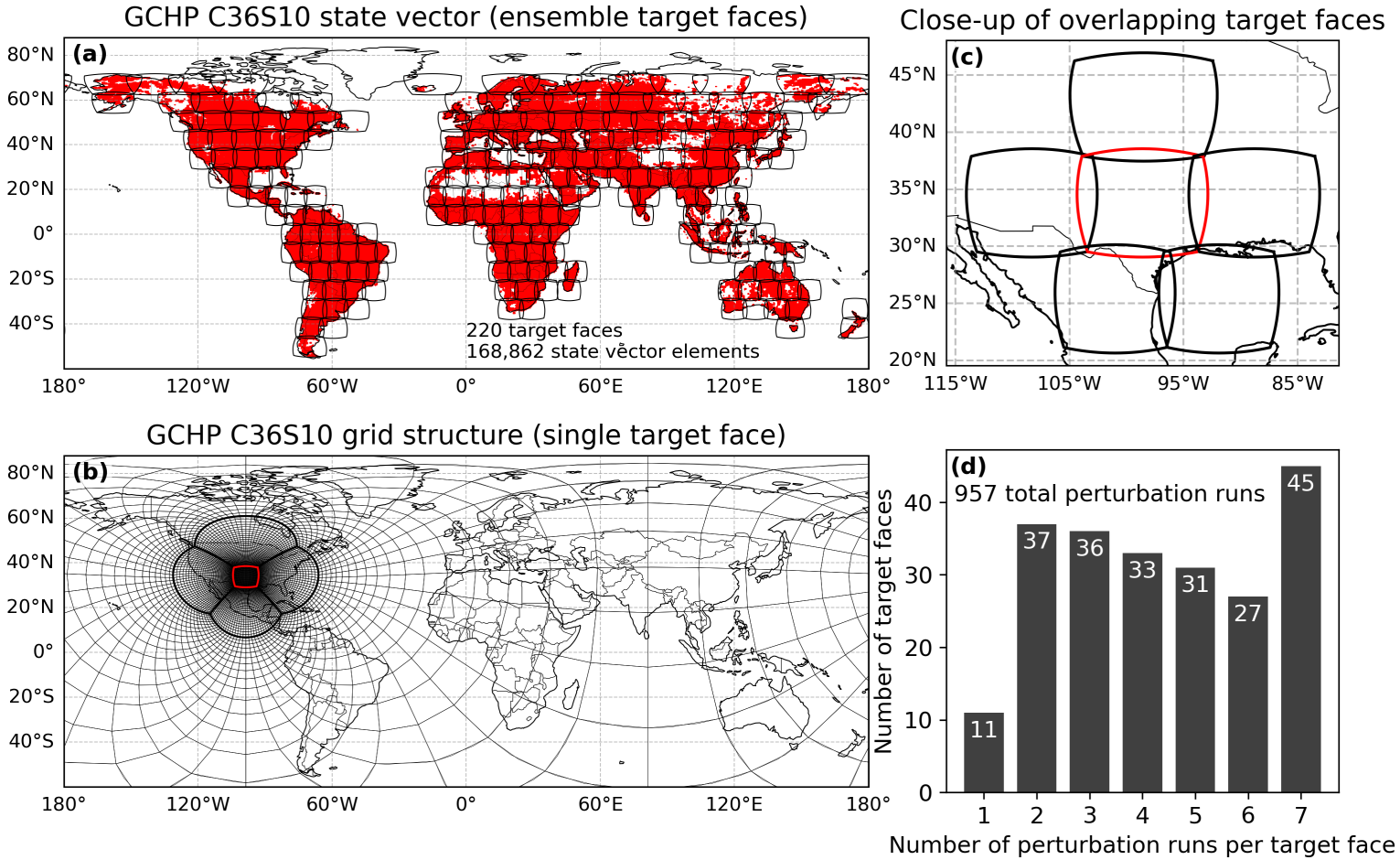

*Martin et al., GMD 2022*

Global concentration response from single emission plume sampled at TROPOMI observations



# Stretched-grid GCHP for global fine-resolution Jacobian sensitivity matrix

## New practical capability of the methane inversion at global 25-km resolution




Ensemble stretched-grid GCHP

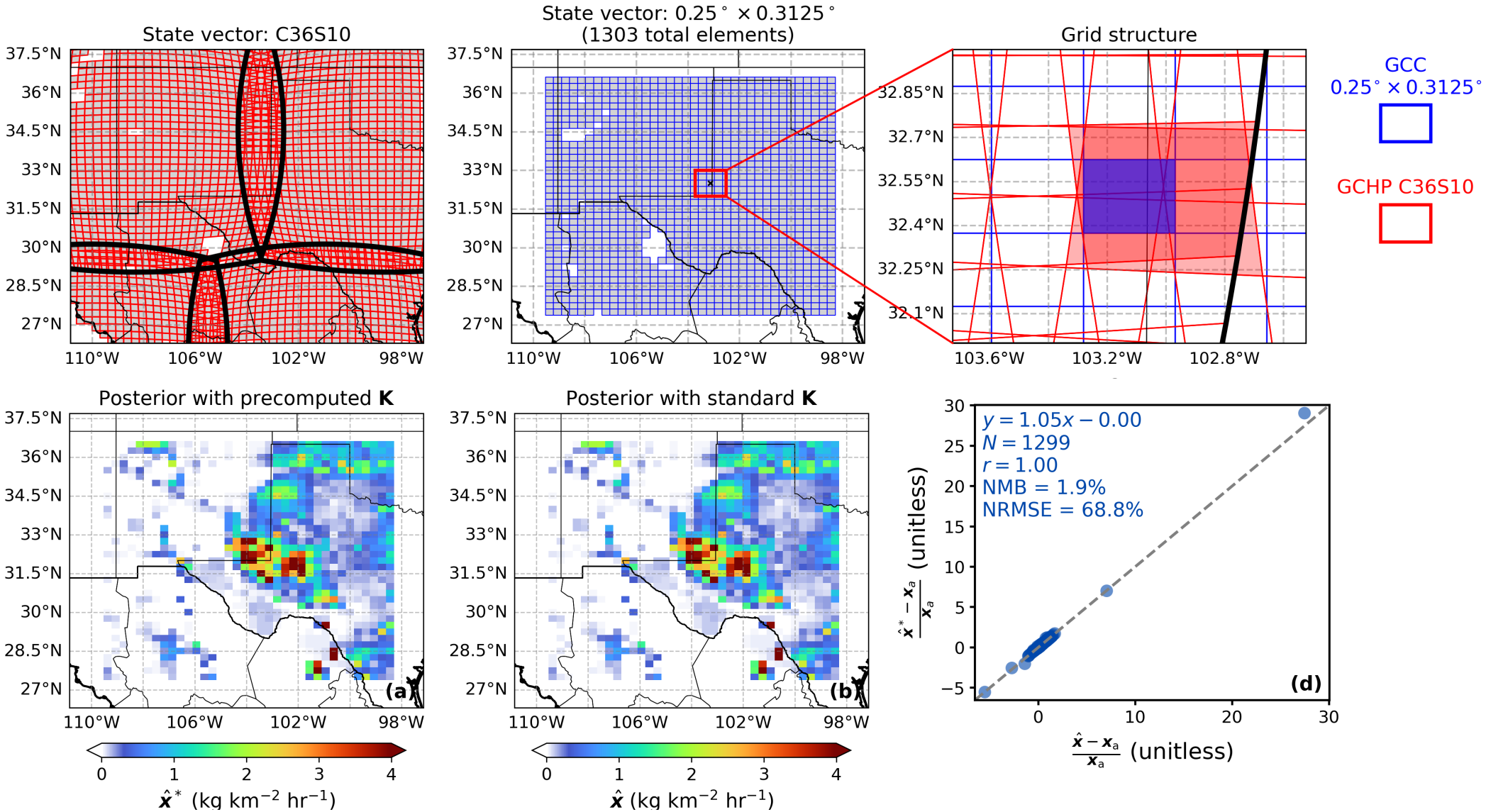
- 220 target faces
- 168,862 state vector elements
- 957 total perturbation runs

BUT

- x100 less computational cost
- x100 less storage requirement

(0.5 TB at C36S10 vs. 50 TB at C360) for full-year Jacobian archive than regular uniform C360.

# Conservative remapping of C36S10 Jacobian archive enables 0.25°×0.3125° regional IMI inversions anywhere at 10x less compute time



# Take-aways

- Stretched-grid GCHP enables the construction of Jacobian at global 25-km resolution with x100 less computational cost and x100 reduced storage requirement
- Using archived global 25-km precomputed Jacobian from stretched GCHP reproduces the posterior estimate using standard IMI with computational cost reduced by ~90%
  => Low-cost regional inversion over any region and any period within the archiving year.

- Next step:
  - Constructing and archiving global 25-km Jacobians from stretched GCHP on AWS
  - A global full-year 25-km inversion of TROPOMI methane observations in 2024
  - Application of ML to extend global 25-km Jacobians to other years